COLOR SUPERFLUIDITY AND CHIRAL SYMMETRY BREAKDOWN IN DENSE QCD MATTER

JIŘÍ HOŠEK

Institute of Nuclear Physics, Řež near Prague, CZ 250 68, Czech Republic

We describe the interplay of two nonperturbative phenomena which should take place in the chirally invariant deconfined phase of QCD matter at finite density and $T=0$: (i) Cooper-pair quark-quark ground-state condensation in appropriate channels should yield exotic sorts of color superfluidity, and (ii) quark-antiquark ground-state condensation should yield spontaneous breakdown of chiral symmetry. We briefly review the main recent achievements in the subject, and present a field-theory formalism which enables to deal with both above mentioned types of condensates simultaneously.

1.Introduction

Long ago a great colleague of ours theoretically speculated that "by convention there is color, by convention sweetness, by convention bitterness, but in reality there are atoms and space." (Democritus, 400 B.C.) Rather ironically, what Democritus called conventions we call reality today. At this conference it is appropriate to paraphrase him as :"By convention there are atoms, by convention mesons, by convention light nuclei, but in reality there are colors, flavors and space":

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}F_{a\mu\nu}F_a^{\mu\nu} + \sum_{\text{F}=1}^{n_{\text{F}}} \overline{\psi}_{\text{F}}(i\not{D} - m_{\text{F}} + \mu_{\text{F}}\gamma_0)\psi_{\text{F}} \; . \qquad (1)$$

Starting from (1) I will discuss some properties of a system of two massless ($m_{\text{F}} = 0$) up and down flavors ($n_{\text{F}} = 2$) of the colored quarks (color indices of quarks are suppressed in (1)) in the deconfined phase characterized by large baryon number density, and zero temperature. I will call the light flavors conservatively up and down although it would be more elegant to call them sweet and bitter. By construction the subject belongs to nuclear physics, and it became rather hot recently.

At high baryon number densities, for example in cores of the neutron stars, hadrons overlap. Due to asymptotic freedom the QCD matter should behave as a weakly interacting quark-gluon plasma[1]. Restricting ourselves to temperatures low enough that the thermal wavelength becomes comparable with interparticle spacing, we can expect behavior of the system characteristic of a genuine quantum Fermi liquid.

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What are the interactions which determine the behavior of the deconfined colored quark and gluon excitations in this phase ? First of all, there are no experimental data, either real or the lattice ones which would provide check of our considerations. From the theoretical point of view we argue as follows : First, the electric components of gluons acquire perturbatively a mass due to Debye screening [1]. This effect gives rise to an effective fourquark interaction of the form

$$\mathcal{L}^{(\mathrm{e})} = G_{\mathrm{e}}(\overline{\psi}\gamma_0 \frac{1}{2}\lambda_a \psi)^2 \,, \tag{2}$$

where $G_{\mathrm{e}}^{-1/2} \sim 300\mathrm{MeV}$. Second, the magnetic components of gluons can be considered as weak external perturbations and neglected in the lowest approximation. They become massive nonperturbatively in the next step due to the dynamical Higgs mechanism. This effect gives rise to an effective fourquark interaction

$$\mathcal{L}^{(\mathrm{m})} = G_{\mathrm{m}}(\overline{\psi}\gamma_i \frac{1}{2}\lambda_a \psi)^2 \,, \tag{3}$$

where $G_{\mathrm{m}} \sim G_{\mathrm{e}}$. Third, at not very large densities there is an effective fourquark interaction originating from quark couplings with the instanton zero modes [4,5] :

$$\mathcal{L}^{(\mathrm{i})} = G_{\mathrm{i}}[(\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma_5\vec{\tau}\psi)^2 - (\overline{\psi}\vec{\tau}\psi)^2 - (\overline{\psi}i\gamma_5\psi)^2] \,, \tag{4}$$

again with $G_{\mathrm{i}} \sim G_{\mathrm{e}}$. This interaction has the important and desirable property of breaking explicitly the global axial U(1) symmetry. Fourth, there are no gluons at zero temperature. Fifth, any massive dynamically generated collective excitation gives rise to a local fourquark interaction the exact form of which cannot be determined a priori. It follows that the dynamics of QCD matter in the deconfined phase at low T is governed by an effective local fourquark $\mathrm{SU(2)_L} \otimes \mathrm{SU(2)_R}$ chirally invariant Lagrangian in which the color $\mathrm{SU(3)_c}$ symmetry is treated as global :

$$\mathcal{L}_{\mathrm{eff}} = \overline{\psi}(i\,\slashed{\partial} + \mu\gamma_0)\psi + \mathcal{L}^{(\mathrm{e})} + \mathcal{L}^{(\mathrm{m})} + \mathcal{L}^{(\mathrm{i})} + \dots \tag{5}$$

On the first sight all these interactions seem to be irrelevant in Wilson's sense when scaling the momenta towards Fermi surface. One is inclined to expect that the system will indeed behave as a (relativistic) Landau Fermi liquid. The argument contains a potential loophole[13] : If any of the considered interactions contains an attraction between two quarks at the Fermi surface with momenta $\vec{p}_{\mathrm{F}}$ and $-\vec{p}_{\mathrm{F}}$, respectively, then this very component becomes marginal, and leads to a phase transition. The effect is well known in nonrelativistic Fermi systems as Cooper instability. It is responsible for the BCS superconductivity, and for the superfluidity of $^3\mathrm{He}$.

We are then faced with an important question : Do the quark interactions in the deconfined low-T phase contain "dangerous" quark-quark attractive channels ? The answer is yes, even in the naive perturbative regime : The one-gluon exchange is attractive not only in the color-singlet quark-antiquark channel ($3 \otimes \overline{3} = 1 \oplus 8$), but with a half strength also in the color-antitriplet quark-quark channel ($3 \otimes 3 = \overline{3} \oplus 6$). Hence, we should expect a quark Cooper-pair condensation in attractive channels and, consequently some sort of color superfluidity or superconductivity. With perturbative forces in mind the idea was mentioned already in [1], and elaborated in [2,3]. Recent interest [6,7,8,9,11,12] is concentrated mainly on the interaction (4).

2. Playing with condensates

With an appropriate local fourfermion interaction between quarks carrying spin, isospin and color there can be just four types of superfluid condensates different from zero due to Pauli principle :

$$v_{(1)} = \langle \overline{\psi} A \tau_2 \gamma_5 \psi^{\mathcal{C}} \rangle \,, \tag{6}$$

$$v_{(2)} = \langle \overline{\psi} A \tau_3 \tau_2 \gamma_0 \gamma_3 \psi^{\mathcal{C}} \rangle \,, \tag{7}$$

$$v_{(3)} = \langle \overline{\psi} S \tau_3 \tau_2 \gamma_5 \psi^{\mathcal{C}} \rangle \,, \tag{8}$$

$$v_{(4)} = \langle \overline{\psi} S \tau_2 \gamma_0 \gamma_3 \psi^{\mathcal{C}} \rangle \,. \tag{9}$$

Although we use the convenient Lorentz-covariant notation with $\psi^{\mathcal{C}} = C\overline{\psi}$ where C is the charge-conjugation matrix, it is clear that the only sacred property of the ground state here is the translational invariance. Consequently, we contemplate four distinct ordered phases :

1. Condensate $v_{(1)}$ with the color-antisymmetric Clebsch-Gordan (CG) matrix A chosen as $A^{ab} = i\epsilon^{ab3}$ corresponds to a ground-state expectation value of the order parameter Φ^c. It describes a Lorentz scalar, isosinglet, color-triplet superfluid.

2. Condensate $v_{(2)}$ corresponds to a ground-state expectation value of the order parameter $\Phi^c_{I;\mu\nu}$. It describes a color-triplet superfluid which at the same time behaves as ordinary, as well as flavor, ferromagnet ($\Phi^c_{I;\mu\nu}$ is an isospin one, antisymmetric tensor field).

3. Condensate $v_{(3)}$ with the color-symmetric CG matrix S chosen as $S^{ab} = \frac{1}{3}\delta^{ab} - \frac{1}{\sqrt{3}}(\lambda_8)^{ab}$ corresponds to a ground-state expectation value of the order parameter Φ^{ab}_I. It describes a Lorentz scalar color-sextet superfluid which at the same time behaves as a flavor ferromagnet.

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4. Condensate $v_{(4)}$ corresponds to a ground-state expectation value of the order parameter $\Phi_{\mu\nu}^{ab}$. It describes an isoscalar color-sextet superfluid which at the same time behaves as an ordinary ferromagnet.

Recent papers are devoted predominantly to the first case with some estimates made [6] also of the fourth possibility.

It is important to realize that all terms in (6-9) are invariant with respect to global chiral SU(2) rotations. This implies that they cannot produce the physical, chiral-symmetry-violating quark masses in the deconfined phase. There is, however, a ground-state condensate which can do namely this,

$$\langle \overline{\psi}\psi \rangle \neq 0 \tag{10}$$

and it should be considered together with (6-9). It is well established that the interactions (5) for the numerical values of the couplings above critical and for densities below critical give rise to the quark masses and, by virtue of the Goldstone theorem, also to the massless pions. In fact, spontaneous breakdown of chiral symmetry at finite density yields nontrivial subtleties revealed only recently [6,10].

3. Self-consistent perturbation theory

Quantitative formulation of the physical program described above is the following : Given the effective $\mathrm{SU}(3))_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ invariant Lagrangian of the type (5) with arbitrary couplings G, analyze its spectrum and properties of its deconfined ordered phases characterized by the vacuum condensates (6-10) different from zero. First real insight into the problem was provided by the BCS-like variational calculation [6]. Here I present main steps of a generalization of the field-theoretical approach developed for superconductivity by Nambu [15], and for the spontaneous chiral symmetry breaking in the nucleon-pion system by Nambu and Jona-Lasinio [16]. It gives almost immediately the dispersion laws of true fermionic quasiparticles as exemplified explicitly below . It also provides a systematic way of investigating the gapless collective Nambu-Goldstone excitations corresponding to all symmetries spontaneously broken by the condensates (6-10). This part of the program will be published separately. For illustration, for comparison with the results of others, and in order to keep the formulas relatively simple I present characteristic results assuming that only $v_{(1)}$ and Σ are different from zero and generated by the interaction $\mathcal{L}^{(i)}$. The general formulas will be published separately.

First, the Lagrangian $\mathcal{L}_{\mathrm{eff}} = \overline{\psi}(i\,\partial\!\!\!/ + \mu\gamma_0)\psi + \mathcal{L}^{(i)}$ is split as $\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_0' + \mathcal{L}_{\mathrm{int}}'$.

The unperturbed ground state defined by $\mathcal{L}'_0 = \mathcal{L}_0 - \mathcal{L}_\Delta - \mathcal{L}_\Sigma$,

$$\mathcal{L}'_0 = \overline{\psi}(i\,\slashed{\partial} + \mu\gamma_0)\psi - \frac{1}{2}[\overline{\psi}A\gamma_5\tau_2\Delta\psi^{\mathcal{C}} + H.c.] - \overline{\psi}\Sigma\psi \qquad (11)$$

is supposed to be energetically favorable with respect to the one defined by $\mathcal{L}_0 = \overline{\psi}(i\,\slashed{\partial} + \mu\gamma_0)\psi$. Selfconsistency is achieved by imposing the condition that the lowest-order perturbative contribution of $\mathcal{L}'_{\text{int}} = \mathcal{L}_{\text{int}} + \mathcal{L}_\Delta + \mathcal{L}_\Sigma$ to $\mathcal{L}'_0$, using the propagator defined by $\mathcal{L}'_0$, vanishes. This condition fixes the numerical values of Δ and Σ in terms of the dimensional parameters of the model. Trivial solutions $\Delta = \Sigma = 0$ always exist and correspond to the naive perturbative expansion.

Second, the main trick consists of introducing the field q which is defined as follows:

$$q^a_{\alpha A} = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi^a_{\alpha A}(x) \\ P^{ab}(\tau_2)_{AB}\delta_{\alpha\beta}\psi^{\mathcal{C}}_{\beta bB}(x) \end{pmatrix} \qquad (12)$$

in which $P^{ab} \equiv [e^{i\alpha}A + e^{i\sigma}S]^{ab}$ has the property $AS = 0$, $P^+P = 1$. The field q operates in space of Pauli matrices abbreviated as Γ_i. In terms of q the Lagrangian of the main interest is

$$\mathcal{L}'_0 = \overline{q} \begin{bmatrix} \slashed{p} - \Sigma + \mu\gamma_0 & -\Delta\gamma_5 \\ -\gamma_0(\Delta\gamma_5)^+\gamma_0 & \slashed{p} - \Sigma - \mu\gamma_0 \end{bmatrix} q \equiv \overline{q}S^{-1}(q)q. \qquad (13)$$

Third, in terms of q the interaction Lagrangian (4) becomes

$$\mathcal{L}^{(i)} = G_i[(\overline{q}q)^2 + (\overline{q}\Gamma_3 i\gamma_5\vec{\tau}q)^2 - (\overline{q}\Gamma_3\vec{\tau}q)^2 - (\overline{q}i\gamma_5 q)^2]. \qquad (14)$$

Fourth, the rest is a well defined manual work :

1. Explicit form of

$$S(p) \equiv \begin{pmatrix} I & J \\ K & L \end{pmatrix}$$

   is the following :

$$I = \frac{\slashed{p}_+ + \Sigma}{D_+}\left[1 + \frac{|\Delta|^2}{D}(\slashed{P} + C)(\slashed{p}_+ + \Sigma)\right], \qquad (15)$$

$$K = \frac{\Delta^*}{D}\gamma_5(\slashed{P} + C)(\slashed{p}_+ + \Sigma), \qquad (16)$$

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and analogous formulas hold for L and J, respectively. Symbols in (15) and (16) are defined as follows :

$$p_+^\mu \equiv [(p_0 + \mu), \vec{p}] \,,$$
$$D_+ \equiv (p_0 + \mu)^2 - \epsilon_p^2 \,, \quad \text{with } \epsilon_p^2 = \vec{p}^2 + \Sigma^2 \,,$$
$$P^\mu \equiv [D_+(p_0 - \mu) - |\Delta|^2(p_0 + \mu), (D_+ - |\Delta|^2)\vec{p}] \,,$$
$$C \equiv -(D_+ - |\Delta|^2)\Sigma \,,$$
$$D \equiv [(p_0 + \mu)^2 - \epsilon_p^2]$$
$$\{p_0^2 - [(\mu + \epsilon_p)^2 + |\Delta|^2]\} \{p_0^2 - [(\mu - \epsilon_p)^2 + |\Delta|^2]\} \,. \quad (17)$$

2. The coupled nonlinear equations for Σ and Δ have the form

$$\Sigma - 13iG_{\mathrm{i}} \int \frac{d^4p}{(2\pi)^4} \, tr(I + L) = 0 \,, \tag{18}$$

$$\Delta^* - 8iG_{\mathrm{i}} \int \frac{d^4p}{(2\pi)^4} (K\gamma_5 + \gamma_5 K) = 0 \,. \tag{19}$$

For further progress it is absolutely crucial that the denominator D of the propagator S, which defines the fermionic spectrum, has been found in the factorized form : The p_0 integration can be performed explicitly using Cauchy theorem, while the remaining integral over d^3p can be either cut off at some physical cutoff Λ, or regularized by the formfactor $F(p^2) = [\Lambda^2/(p^2 + \Lambda^2)]^\nu$ as suggested in [6]. In any case : if $\Delta \neq 0, \Sigma \neq 0$ are found by solving (18) and (19), then the lowest-order selfconsistent perturbation theory turns the system of interacting massless quarks into a system of noninteracting massive quasiquarks with the dispersion laws explicitly given by (17).

3. For $\Delta = 0$ the equation (18) is the standard gap equation for the dynamical quark mass in the popular NJL approach [16]. For $\Sigma = 0$ we have verified that the gap equation (4.4) of [6] is obtained in an approximation of keeping only the leading term of the strongest residue in (19).

4. In the vacuum sector ($\mu = 0$) all formulas greatly simplify, and the form of our coupled vacuum gap equations

$$\Sigma - 104iG_{\mathrm{i}} \int \frac{d^4p}{(2\pi)^4} \frac{\Sigma}{p^2 - \Sigma^2 - |\Delta|^2} = 0 \,, \tag{20}$$

$$\Delta^* - 16iG_{\mathrm{i}} \int \frac{d^4p}{(2\pi)^4} \frac{\Delta^*}{p^2 - \Sigma^2 - |\Delta|^2} = 0 \,, \tag{21}$$

is in accord with the formulas of [14].

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4.Outlook

Recent semiquantitative analyses of the deconfined quark low-temperature matter using the realistic numerical values of the parameters reveal the potential relevance of the ordered phases for the heavy-ion collisions. In general, the vacuum engineering with new peculiar macroscopic quantum phases is very promising, and it is worth of being developed theoretically in all possible details. Fortunately, this seems feasible. We find gratifying that the NJL-like effective field-theory treatment is fully justified for the description of the deconfined phase. Of course, it has to be supplemented with a prescription which properly mimics the asymptotic freedom of the underlying QCD.

Acknowledgments

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